Learning and Predicting Chaos using LSTM RNNs

Malvern Madondo and Thomas Gibbons (Advisor)
Department of Computer Information Systems, The College of St. Scholastica, Duluth, MN

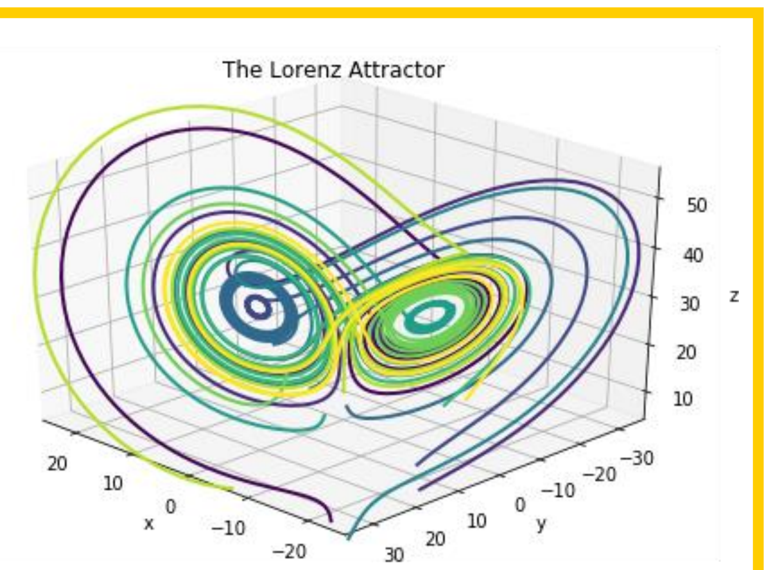
Abstract

In nonlinear dynamical systems, long-term prediction is extremely challenging. Small perturbations in an initial state can grow exponentially in time and result in large differences in a later advanced state - a behavior known as *chaos.* Chaotic systems tend to have sensitive dependence on initial conditions, much like the Butterfly Effect. Recurrent Neural Networks (RNNs) are dynamic and allow for modeling of chaotic behavior. In this paper, we study and investigate the the modeling and prediction abilities of a Long Short-Term Memory (LSTM) recurrent neural network in dynamical systems with chaotic behavior. In particular, we explore the Lorenz System - which comprises of a nonlinear system of differential equations describing two-dimensional flow of a fluid, and describe an architecture that models the systems' behavior.

Introduction

The Lorenz System

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$



$\sigma = 10.0$, $\rho = 28.0$, $\beta = 8/3$

x, y, z – physical properties of the system w.r.t time

Objectives

- Investigate the modeling and prediction abilities of a traditional Recurrent Neural Network (RNN) and a "Long Short-Term Memory" (LSTM) RNN, when the input signal has a chaotic nature.
- Study the effectiveness of both networks in predicting the Lorenz System one-step ahead.

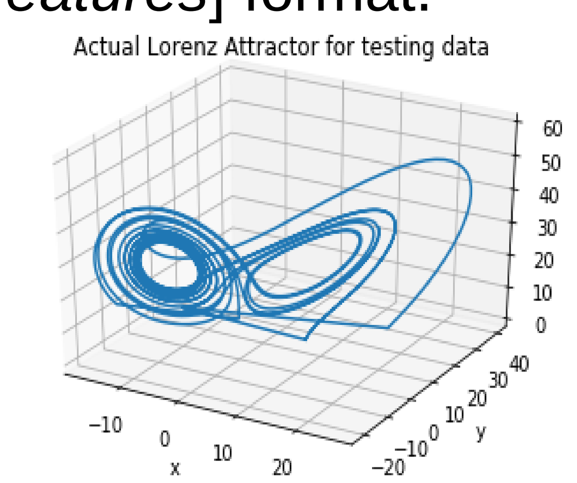
Methodology

- Generate data using the Lorenz System.
- Split the data into training and testing sets.
- Convert the data and fit the neural network models in Keras
- Plot and compare the results for each network.

Experiments

Reshape the data into a [*samples, time steps, features*] format.
Training set has 70% of data and testing, 30%.

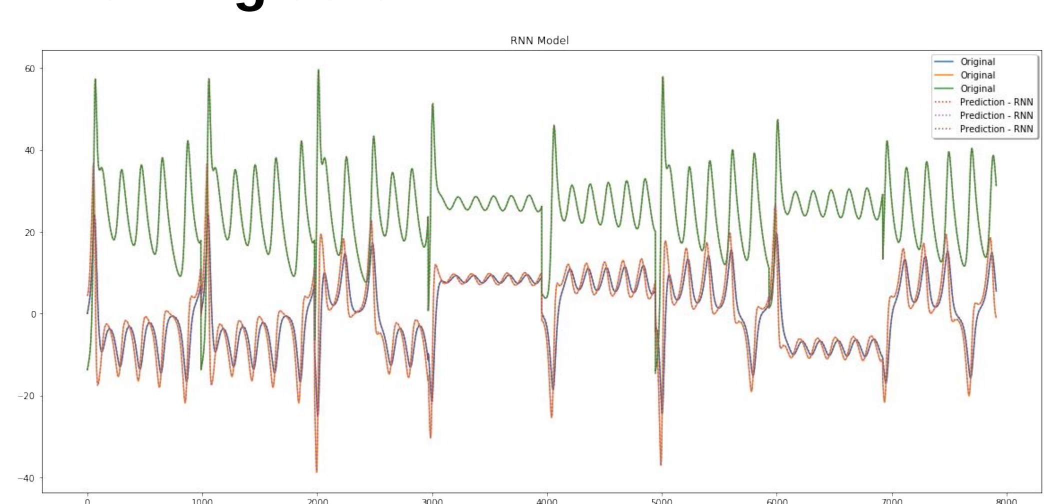
Shape of training input: (7912, 10, 3)
Shape of training output: (7912, 3)
Shape of test input: (3956, 10, 3)
Shape of testing output: (3956, 3)

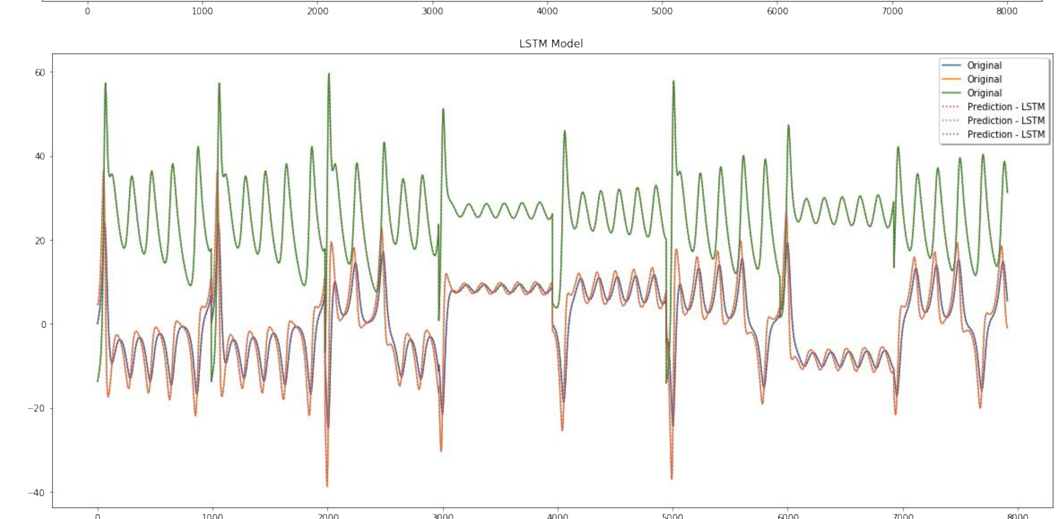


Model Architecture:

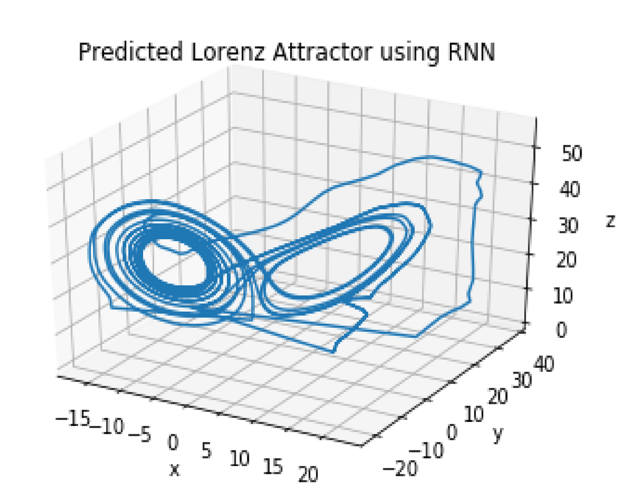
```
model = Sequential()
model.add(LSTM(16, input_shape=(None, 3)))
model.add(Dense(3))
model.compile(loss='mean_squared_error', optimizer='adam')
```

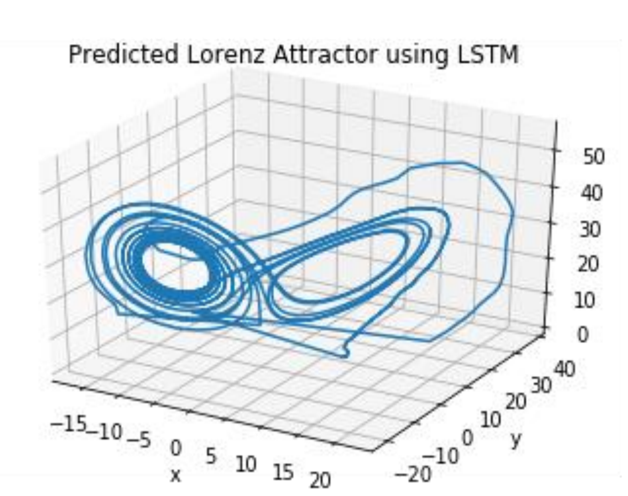
RNN and LSTM Predictions on Training data





RNN and LSTM Models on Testing data





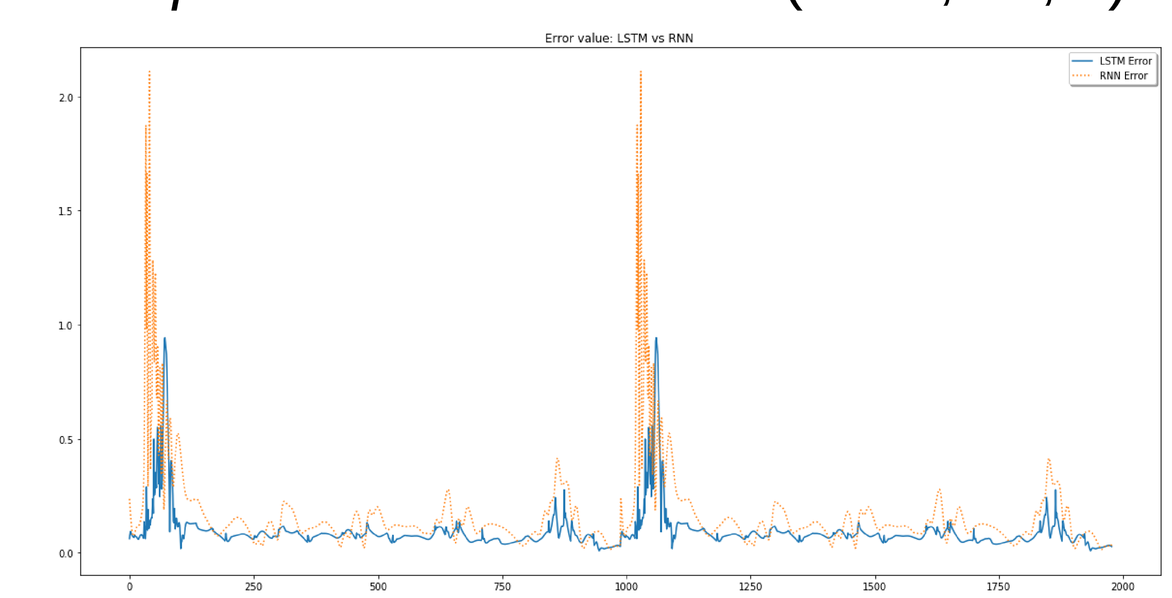
Results

Model	Epochs	Validation Loss
RNN	50	0.2149
LSTM	50	0.1523
RNN	75	0.0722
LSTM	75	0.0202

Analysis

Both the traditional RNN and the LSTM were applied to chaotic data generated from the Lorenz System.
Testing with single starting point and Predicting one-step ahead:

Shape of data: (1, 1000, 3)
Shape of windowed data: (1978, 10, 3)



One-step prediction performance on test data.

	RNN	LSTM
Average Prediction Error	0.1691547258621755	0.09410667635011637

Conclusion

- It is observed that the LSTM network outperforms the traditional RNN due to its ability to learn long term dependencies as the system grows.
- LSTM networks can efficiently model and predict dynamic systems such as the Lorenz System.

References

- Lorenz, Edward N. "Deterministic Nonperiodic Flow". *Journal of the Atmospheric Sciences,* vol.20, 130-141, 1963.
- Woolley, Jonathan W., Agarwal, P. K., and Baker, John. Modeling and prediction of chaotic systems with artificial neural networks. *International Journal for Numerical Methods in Fluids.* 63:989–1004, 2010.
- Zaytar, M.A. Amrani, C.E. Sequence to Sequence Weather Forecasting with Long Short-Term Memory Recurrent Neural Networks. *International Journal of Computer Applications (0975 - 8887). Volume 143 - No.11, June 2016.*