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An Empirical Study of the Vertex Cover Problem

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Abstract

VERTEX COVER is a core graph problem in the class NP-complete. Given a graph with a set of vertices and a set of edges, we find a smallest subset of the vertices that covers all the edges. An edge is considered to be covered if either of its end points is in the cover set. As with all NP-complete problems, there are exponential-time algorithms that give exact solutions, but it is unknown whether there are deterministic polynomial-time algorithms. In this study we contrast and compare the performance of three deterministic algorithms for VERTEX COVER. One algorithm is taken from a recent research paper on phase transitions (also called threshold phenomena) in combinatorial problems. The second algorithm is a depth-first backtracking algorithm with heuristics not found in the first algorithm, and the third is a breadth-first approach. The algorithms will be used to illustrate the threshold phenomena in the problem space, and their performance will be compared with the goal of determining whether threshold behavior is an algorithm-dependent phenomenon, or whether all algorithms indicate phase transition in the same region of the problem space.

The empirical results provide the basis for a general discussion of threshold behavior. Thresholds are typically encountered where the probability of finding a solution in randomly generated problem instances changes rapidly as the instance size grows. The region in which this change occurs is sometimes referred to as the critical region or crossover region. Are thresholds inherent in the problem space or artifacts of algorithms that employ depth-first search? Are algorithms that don't exhibit threshold behavior better or worse than the ones that do? Do some algorithms indicate thresholds in regions other than the crossover region?

This empirical study and the ensuing discussion provide a background for continuing research in combinatorial problems. They also provide an overview of a current research problem in computer science at a level that is appropriate for use in upper-division undergraduate courses in algorithms.